Tutorials - Week 4 - Sep 26

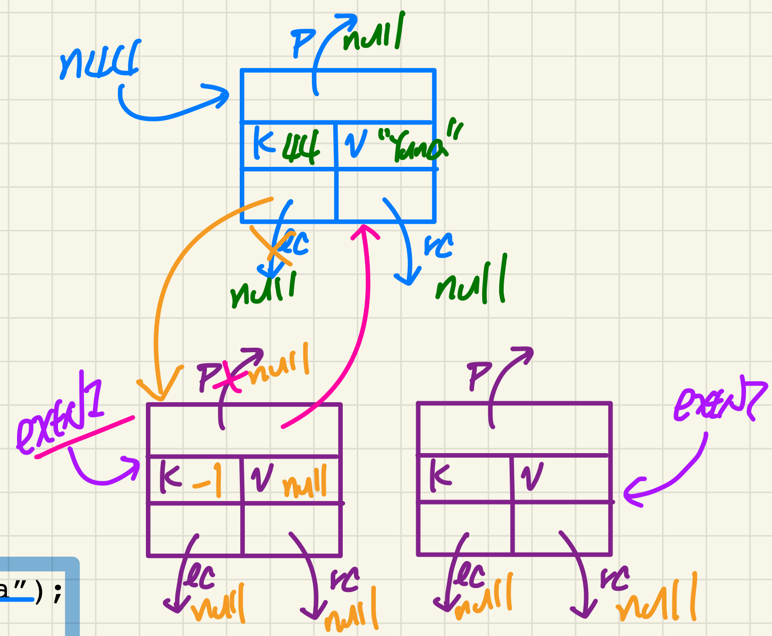
Self-Balancing BST

Task Preview: Right Rotation in Java
BST Deletion: Case 4
Trinode Restructuring after Deletions

Exercise: BST Construction

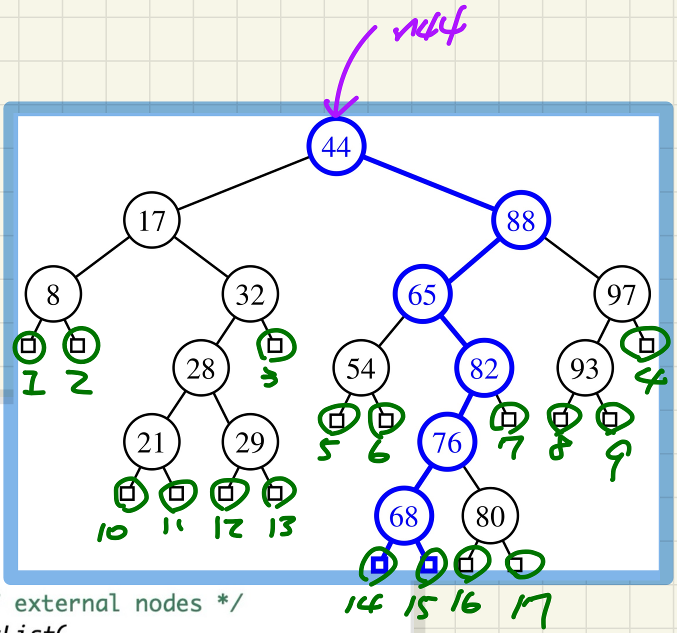
```java
public class BSTNode<E> {
   private int key;
   private E value;

   private BSTNode<E> parent;
   private BSTNode<E> left;
   private BSTNode<E> right;

   . . .

   public void setLeft(BSTNode<E> left) {
      this.left = left;
      left.setParent(this);
   }

   public void setRight(BSTNode<E> right) {
      this.right = right;
      right.setParent(this);
   }

}
```

Study: *constructExampleTree*




```java
BSTNode<String> n44 = new BSTNode<>(44, "Yuna");
BSTNode<String> extN1 = new BSTNode<>();
BSTNode<String> extN2 = new BSTNode<>();
n44.setLeft(extN1);
n44.setRight(extN2);
```

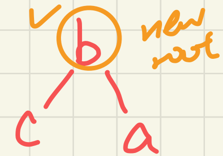
Exercise: BST In-Order Traversal

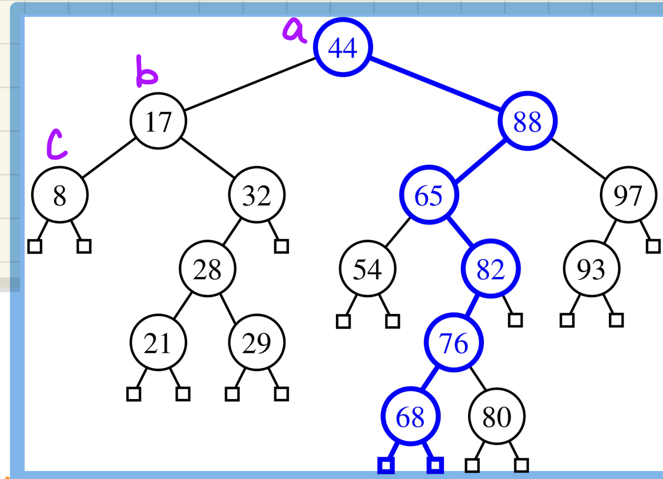


root of subtree

```
@Test
public void test_bst_in_order_traversal() {
    constructExampleTree();

    BSTUtilities<String> u = new BSTUtilities<>();
    ArrayList<BSTNode<String>> inOrderList = u.inOrderTraversal(n44);
    assertTrue(inOrderList.size() == 16 + 17); /* 16 internal nodes + 17 external nodes */
    ArrayList<BSTNode<String>> expectedOrder = new ArrayList<>(Arrays.asList(
                    extN1, n8, extN2,
                n17,
                    extN10, n21, extN11, n28, extN12, n29, extN13, n32, extN3,
            n44,
                    extN5, n54, extN6, n65, extN14, n68, extN15, n76, extN16, n80, extN17, n82, extN7,
                n88,
                    extN8, n93, extN9, n97, extN4
    ));
    assertEquals(expectedOrder, inOrderList);
}
```

Exercise: Trinode Restructuring via a Right Rotation






```java
@Test
public void test_bst_right_rotation_1() {
    constructExampleTree();

    BSTUtilities<String> u = new BSTUtilities<>();
    u.rightRotate(n44, n17, n8);

    ArrayList<BSTNode<String>> inOrderList = u.inOrderTraversal(n17);
    assertTrue(inOrderList.size() == 16 + 17); /* 16 internal nodes + 17 external nodes */
    ArrayList<BSTNode<String>> expectedOrder = new ArrayList<>(Arrays.asList(
            extN1, // T1
            n8, // c
            extN2, // T2
            n17, // b
            extN10, n21, extN11, n28, extN12, n29, extN13, n32, extN3, // T3
            n44, // a
            extN5, n54, extN6, n65, extN14, n68, extN15, n76, extN16, n80, extN17, n82, extN7, n88, extN8, n93, extN9, n97, extN4 // T4
    ));
    assertEquals(expectedOrder, inOrderList);
}
```

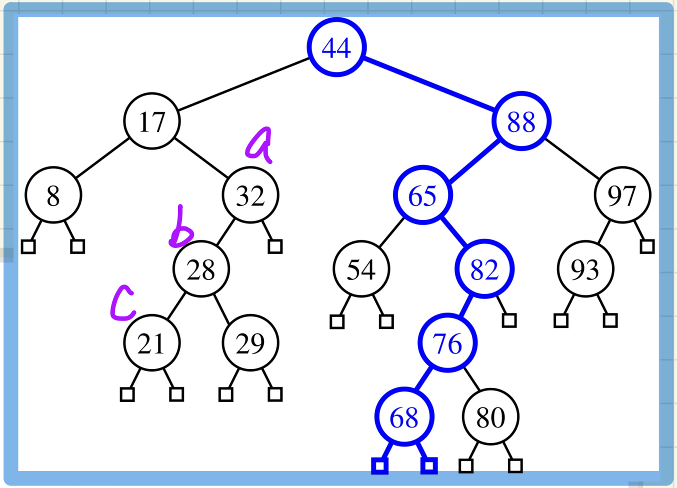
a b c

after R-rotation, the new root

new root

Identical to the T.O.a. seq. before rotation.

Exercise: Trinode Restructuring via a Right Rotation




```java
@Test
public void test_bst_right_rotation_2() {
    constructExampleTree();

    BSTUtilities<String> u = new BSTUtilities<>();
    u.rightRotate(n32, n28, n21);
                  a    b    c

    ArrayList<BSTNode<String>> inOrderList = u.inOrderTraversal(n44);
    assertTrue(inOrderList.size() == 16 + 17); /* 16 internal nodes + 17 external nodes */
    ArrayList<BSTNode<String>> expectedOrder = new ArrayList<>(Arrays.asList(
            extN1, n8, extN2, n17,
            extN10, // T1
            n21, // c
            extN11, // T2
            n28, // b
            extN12, n29, extN13, // T3
            n32, // a
            extN3, // T4
            n44, extN5, n54, extN6, n65, extN14, n68, extN15, n76, extN16, n80, extN17, n82, extN7, n88, extN8, n93, extN9, n97, extN4
    ));
    assertEquals(expectedOrder, inOrderList);
}
```

the rotation does not alter the root of the entire tree.

BST Operation: Cases of Deletion

n1 .. n2 ... ~~n3~~ ~~p~~ n4 ... n5 .. n6

n3 ① replace p by n3

② remove n3

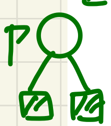
To *delete* an ***entry*** (with **key** *k*) from a BST rooted at ***node*** *n*:

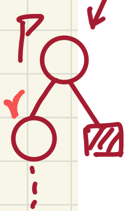
Let node ***p*** be the return value from `search(n, k)`.

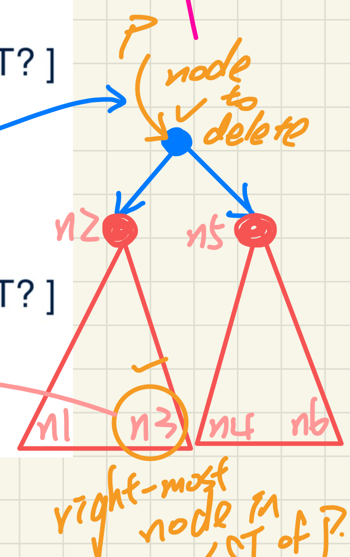
root

key of entry to be deleted.

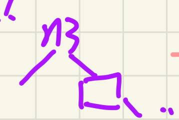
- **Case 1**: Node ***p*** is ***external***.
  k is not an existing key ⇒ Nothing to remove
- **Case 2**: Both of node ***p***'s child nodes are ***external***. (internal)
  No "orphan" subtrees to be handled ⇒ Remove ***p*** [Still BST?]
- **Case 3**: One of the node ***p***'s children, say *r*, is ***internal***.
  - *r*'s sibling is ***external*** ⇒ Replace node ***p*** by node *r* [Still BST?]
- **Case 4**: Both of node ***p***'s children are ***internal***.
  - Let *r* be the ***right-most internal node p's LST***.
    ⇒ *r* contains the ***largest key s.t. key(r)*** < ***key(p)***.
    Exercise: Can *r* contain the ***smallest key s.t. key(r)*** > ***key(p)***?
  - Overwrite node ***p***'s entry by node *r*'s entry. [Still BST?]
  - *r* being the ***right-most internal node*** may have:
    ◇ Two ***external child nodes*** ⇒ Remove *r* as in **Case 2**.
    ◇ An ***external, RC*** & an ***internal LC*** ⇒ Remove *r* as in **Case 3**.







Q. Is it possible?

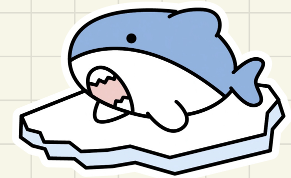


n3

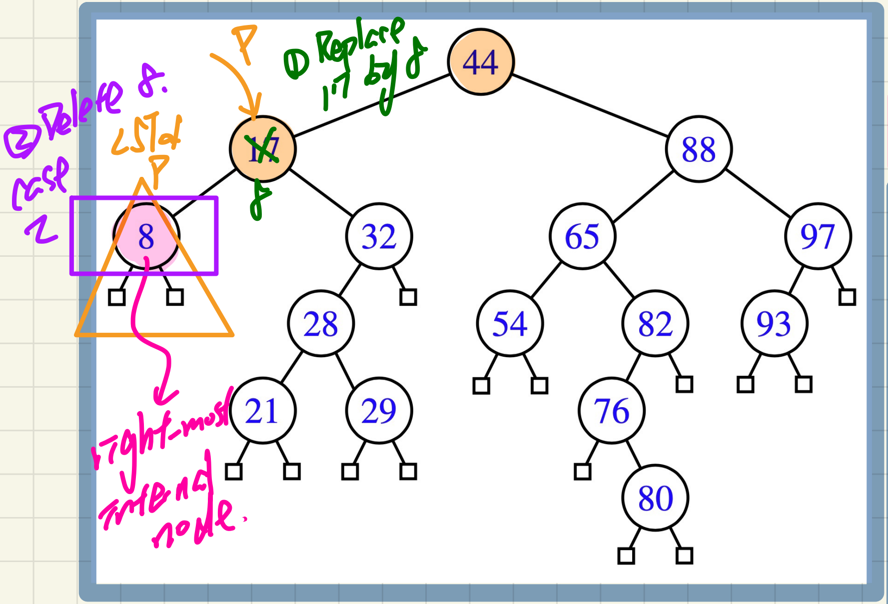
Case 1

Case 2

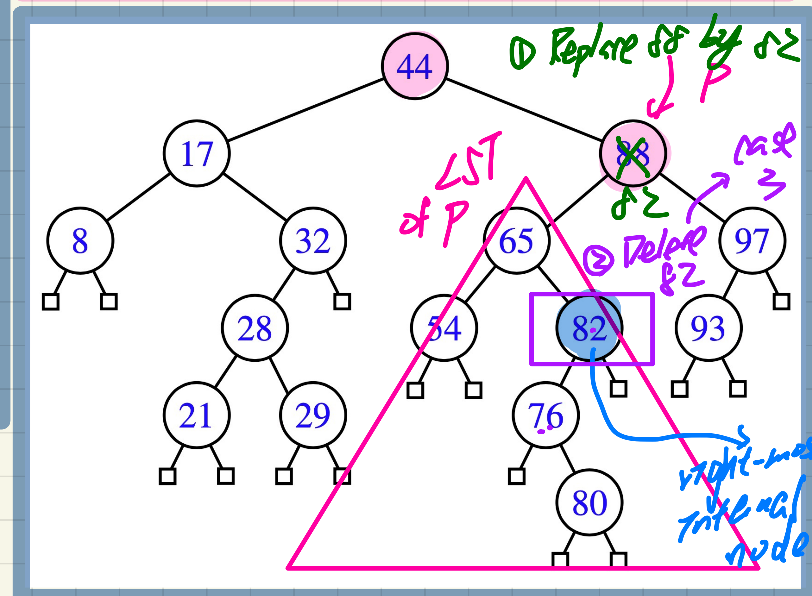
Visualizing BST Operation: Deletion



Case 4.1: Delete Entry with Key 17



Case 4.2: Delete Entry with Key 88



After Deletion: Continuous Trinode Restructuring

- *Recall*: ***Deletion*** from a BST results in removing a node with zero or one ***internal*** child node.
- After ***deleting*** an existing node, say its child is *n*: internal child

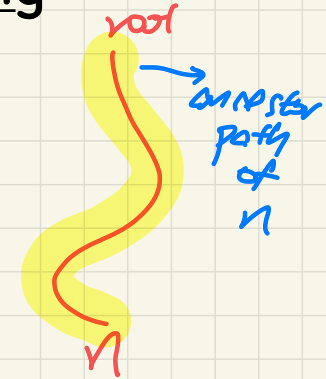
  Case 1: Nodes on *n*'s ***ancestor path*** remain ***balanced***. ⇒ No rotations

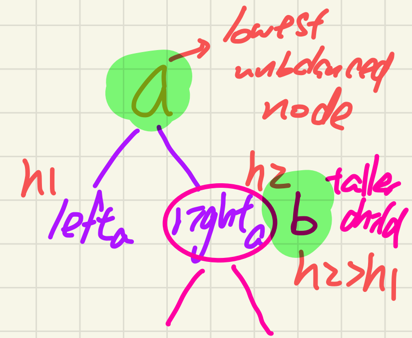
  Case 2: At least one of *n*'s ***ancestors*** becomes ***unbalanced***.

  1. Get the **first**/**lowest** ***unbalanced*** node *a* on *n*'s ***ancestor path***.
  2. Get *a*'s ***taller*** child node *b*. [$b \notin$ *n*'s ***ancestor path***]
  3. Choose *b*'s child node *c* as follows:
     - *b*'s two child nodes have ***different*** heights ⇒ *c* is the ***taller*** child
     - *b*'s two child nodes have ***same*** height ⇒ *a*, *b*, *c* slant the ***same*** way
  4. Perform rotation(s) based on the ***alignment*** of *a*, *b*, and *c*:
     - Slanted the ***same*** way ⇒ ***single** rotation* on the **middle** node *b*
     - Slanted ***different*** ways ⇒ ***double** rotations* on the **lower** node *c*
- As *n*'s ***unbalanced ancestors*** are found, keep applying **Case 2**, until **Case 1** is satisfied. [***rotations***]

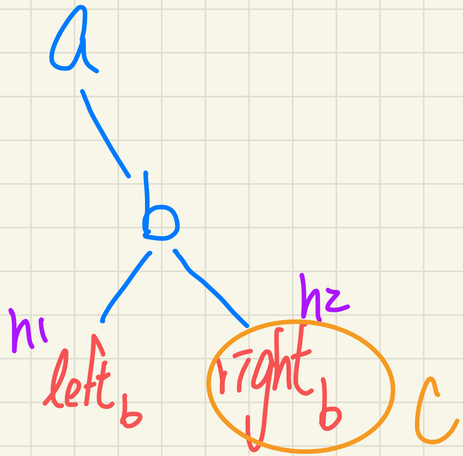
there might be multiple trinode restructuring steps to perform

~~O(h)~~ O(h) = O(log n).





(case 1)

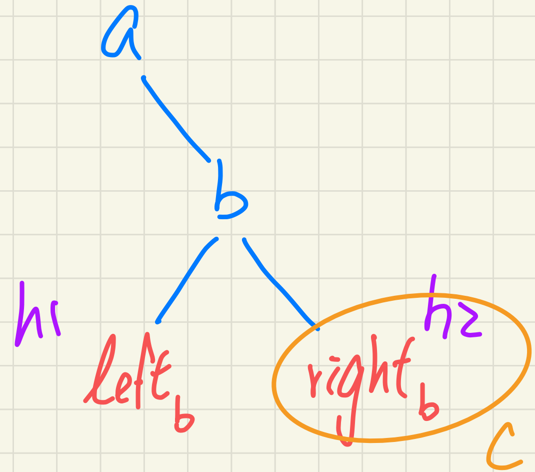


$h_1 \neq h_2$

$\Rightarrow$ choose the taller child.

say: $h_2 > h_1$

(case 2)



$h_1 = h_2$

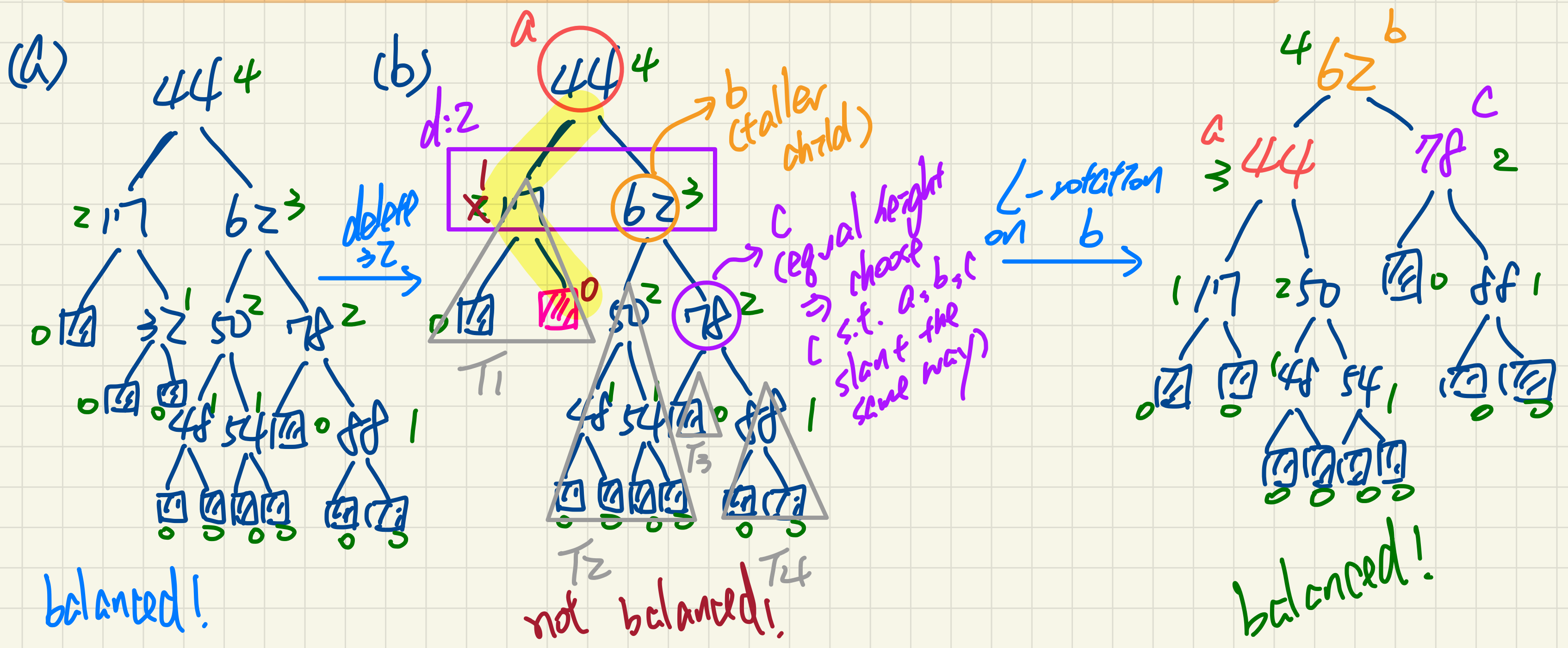
$\Rightarrow$ choose the child that will make a, b, c slant the same way

Trinode Restructuring after Deletion: Single Rotation

- Insert the following sequence of keys into an empty BST:

  <44, 17, 62, 32, 50, 78, 48, 54, 88>

- Delete 32 from the BST.



Trinode Restructuring after Deletion: Multiple Rotations

- Insert the following sequence of keys into an empty BST:

  <50, 25, 10, 30, 5, 15, 27, 1, 75, 60, 80, 55>

- Delete 80 from the BST.